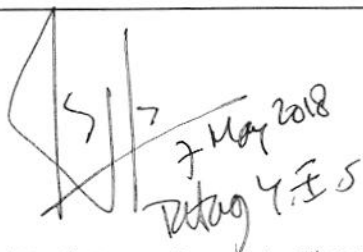




Flexibility in Mathematics Education

Taipei, Taiwan May 7-11, 2018



Cite as:

Hsieh, F.-J. (Ed.), (2018). *Proceedings of the 8th ICMI-East Asia Regional Conference on Mathematics Education*, Vol 2, Taipei, Taiwan: EARCOME.

Website: <http://earcome8.math.ntnu.edu.tw>

The proceedings are also available online

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ISBN 978-986-05-5784-8

Cover design: Chia-Hsin Lin

PREFACE

EARCOME is an ICMI regional conference with an aim to help the growth of research communities in mathematics education for countries in and nearby the East Asian region.

EARCOME 8 convenes in Taipei, Taiwan from May 7 to May 11 in 2018. This conference welcomes participants not only from the East Asian region but also from around the globe, especially scholars who have interests in finding collaborative partners or special works to share with East Asian scholars. On behalf of the conference Local Organizing Committee I am very excited to welcome you to Taipei. More than 350 people will attend the conference, representing about 40 countries.

EARCOME 8 is a collaborative effort of teams from the Department of Mathematics of National Taiwan Normal University, the Taiwan Association for Mathematics Education, and the Shi-Da Institute for Mathematics Education. The conference is possible only with the time and energy put in by three committees: the International Program Committee (IPC), the Advisory Committee (AC), and the Local Organizing Committee (LOC). I acknowledge the enormous efforts of these committees. I especially address my thanks to the conference secretary, Ms. Shiao-Huei Li. Without of her, EARCOME 8 will not be possible. She has devoted all her time to bringing the conference to practice. I especially want to give my special thanks to each of you. Thank you for contributing to the conference and making your journey to EARCOME8 in Taipei.

"Flexibility in Mathematics Education" has been chosen as the theme of the conference. The completion of the EARCOME 8 conference proceedings shows a best example of flexibility in mathematics education. Reviewers, and many authors, from various countries who worked on flexibility reviewed one another's papers with a mind of flexibility. The decision group implemented a sense of flexibility to accept not only research papers, but also those embodied descriptive, analytic, or persuasive natures, if they contained significant educational viewpoints. Authors whose papers were accepted to publish in the conference proceedings were allowed to select either to publish the abstracts only or the full papers as you might see in the conference proceedings.

Papers in the EARCOME 8 proceedings deliberate their points relating to flexibilities in mathematics education, explicitly or implicitly from different dimensions in various contexts. Some attempted to conceptualize flexibility in mathematics education and to clarify the attributes, connotation, and denotation of it. Others address it by the types, phases, degrees, and ranges. Flexibility in mathematics education has also been allied to epistemology, philology, psychology, science, and technology. There are terms often associated with it such as creativity, multiplicity, variation, connection, thinking, changeable, transferable, and adaptation in the conference proceedings.

Papers focused on the student's and the teacher's sides embrace the areas of learning materials (such as using comics in textbooks), activities (in and out of the classroom), problem solving (using alternative ways or in different formats), evaluation (measuring flexible competences), modelling (integrating and switching among different domains), and curriculum (reforming and transforming). Papers focused on the teacher education side consider lesson plans, teaching competences, and professional development in many areas such as in modelling or in ICT activities.

The conference proceedings includes some renowned educational trends, such as the STEM and STEAM (Science, Technology, Engineering, Art, and Mathematics) curriculums and the Big Data concept; these enterprises inherit a requirement of flexibility in the design, generation, or construction processes, and in the collaborative nature of these processes. The proceedings also includes some initiative educational trends, such as the MGA (mathematics grounding activities) which implements investigation, creation, and gamification in the levels of learners, teachers and teacher educators at the outside and inside of school settings.

The Local Organizing Committee of ICMI-EARCOME 8 brought the concept of flexibility in mathematics education to front by choosing it as the theme of the conference. The reasons for the choice are posted in the conference website: "Flexibility is highly related to creativity, multiplicity, and adaptation. In the current era, rapid changes in economy, environment and society have been facilitated by the rapid development of technology and engineering. Flexibility in mathematical thinking, problem solving, teaching methods, evaluation, teacher education and mathematics education research is a key to empowering learners, teachers, educators and researchers to tackle the complexity and uncertainty, and to giving them the capacity and motive to change in the innovative era." The conference ended up with a fruitful production about flexibility in mathematics education that you can find by reading through the two volumes of this conference proceedings.

I thank all our colleagues and friends who chose to present their work and send their papers for the proceedings to materialize. I commend the invited speakers for their enthusiasm in sharing their ideas, even if it meant they took the extra time to write when they could have easily declined the request to do so. I am also grateful to the TSG presenters who eagerly sent their full paper for review. Blind reviews were implemented; I thank all the reviews, without of them, the book will not be able to complete in this quality.

I hope you take the chance to read each and every paper. I assure you that they are worth your time.

Long Jui Shieh 謝聖瑞

ICMI-EARCOME 8 Conference Chair

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AN INNOVATIVE TRAINING MODEL FOR SUPPORTING IN-SERVICE TEACHERS' UNDERSTANDING ON PROBLEM-SOLVING KNOWLEDGE FOR TEACHING

Tatag Yuli Eko Siswono, Universitas Negeri Surabaya, Indonesia

Ahmad Wachidul Kohar, Universitas Negeri Surabaya, Indonesia

Sugi Hartono, Universitas Negeri Surabaya, Indonesia

Abdul Haris Rosyidi, Universitas Negeri Surabaya, Indonesia

Introduction

The insignificant improvement of Indonesian students in performing problem-solving (PS) tasks like in PISA and TIMMS (Mullis et al, 2012; OECD, 2015) and weaknesses on teachers' mathematical problem solving knowledge for teaching (MPSKT) as we reported in the previous article (Siswono et al., 2016, 2017) calls for teacher professional development which focus on developing teachers' MPSKT (Chapman, 2015). Thus, the aim of this study is to report the learning sequences of a training teacher model designed by the authors to support secondary mathematics teacher learn problem-solving content knowledge (PSCK) and pedagogical problem-solving knowledge for teaching (PPSK). In this study, we focus on developing teachers' understanding on three types of PSCK: knowledge of problem, model of problem solving (PS processes and strategies), and problem posing (before, during, and after PS), as well as the PPSK: instructional practices of PS.

Table 1 describes the categories of problem solving knowledge into three main categories, namely problem solving content knowledge (PSCK), pedagogical problem-solving knowledge (PPSK), and knowledge of affective factors relating to problem-solving teaching. PSCK includes four categories. First, knowledge of the meaning of the problem. Teachers need to understand issues based on their structure and goals in order to guide students find solutions including an understanding of the types of tasks, such as tasks with the potential to develop mathematical creativity in problem solving; tasks that allow various solution strategies, and open-ended task. Second, knowledge of mathematical problem-solving proficiency including conceptual understanding of mathematical concepts, procedures and relations, an understanding of heuristic strategies and special strategies and when and how to use them in solving math problems. Third, knowledge of PS. This knowledge is necessary, for example, to understand models of understanding of the problem-solving process. These problem-solving models have been formulated by experts such as Schoenfeld (1985): analysis, design, exploration, implementation and verification; and Polya (1973): understanding the problem, devising plan/strategies, carrying out the plan, and looking back. Regardless of the problem-solving model being studied, Chapman asserts that teachers need to have conceptual and procedural understanding of the problem-solving models in order to understand the stages required by a problem solver and the thinking process involved in finding the solution of the current problem. Fourth, knowledge of math problem submission. This knowledge refers to Silver's (1994) opinion which poses a posing problem as an activity to generate a new problem and reformulate the given problem.

Table 1. Mathematical Problem-solving knowledge for teaching (MPSKT) (Chapman, 2015)

Type of knowledge	Knowledge
Problem solving content knowledge	Mathematical problem solving proficiency
	Mathematical problems
	Mathematical problem solving
	Problem posing
Pedagogical problem solving knowledge	Students as mathematical problem solvers
	Instructional practices for problem solving
Affective factors and beliefs	

PPSK, on the other hand, consists of two subcategories. First, knowledge of students as problem solvers. This knowledge, according to Chapman, will help teachers develop appropriate students' problem-solving skills. In general, this knowledge includes knowledge of students' problem-solving difficulties, successful problem-solving characteristics, and the process of thinking in problem solving. However, to support the development of student problem-solving skills, the current learning perspective shows that this knowledge can be understood from a student's point of view with a focus on building what students know and can do in trying to solve problems in their own way. Second, knowledge of teaching problem solving. According to her, teachers need to understand learning practices that develop strategies and metacognition of students in solving problems. Teachers must have strategic competencies to address problem-solving challenges during teaching. Teachers need to also understand the different implications of different approaches chosen, whether they can be useful or not, and if not, what factors might be causing them. Teachers should be able to decide when and how to intervene, when to provide help and how forms of assistance can support student success while ensuring that they maintain their settlement strategies. In addition, teachers need to know what to do when students are stuck or are using an unproductive or time-consuming approach.

Meanwhile, knowledge of affective factor related to problem-solving consists of motivation, interest, self-confidence, anxiety, perseverance, and student beliefs. Chapman argues that knowledge of the affective aspects of students is important because it plays an important role in problem solving. Knowledge of these factors can help teachers to illustrate and support student problem-solving abilities based on findings that are appropriate to these factors. Besides, teachers also need to know their own beliefs about mathematics and its learning and teaching as well as their impacts to students' successful at PS. In sum, Chapman emphasized the importance of teacher attitudes that can assist students in problem-solving activities.

Method

We involved 90 teachers in the training model in three programs (each 40, 30, and 20 respectively). The participants of the first two groups were inservice secondary teachers in Mojokerto and Jember city, Indonesia, while the other group was primary teachers who were taking master of education programme at Universitas Negeri Surabaya, Surabaya,

Indonesia. The design principles of the training model follow the framework of MPSKT proposed by Chapman (2015) and the modification of Steinbring's model (1998) by Zaslavsky (2009) as follows (Figure 1).

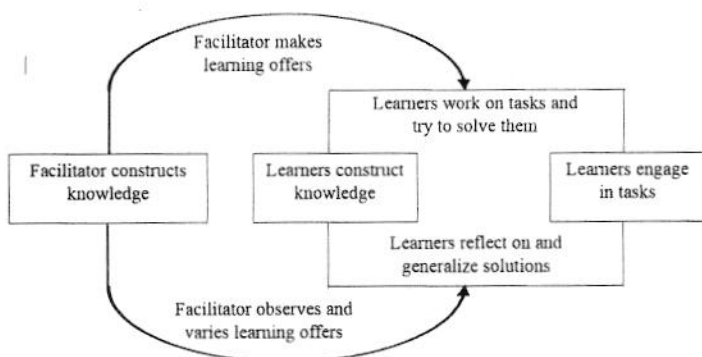


Figure 1. Modification of Steinbring's (1998) Model of Teaching and Learning Mathematics by Zaslavsky (2009)

Figure 1 shows a model of training teachers from two loops, in which one denotes learning by reflection from the teacher and the second shows the facilitator's learning with the reflection and observation of the process undertaken by the teacher. In this training model, when teachers are engaged in the task and build knowledge about the MPSKT, the facilitator will reflect the learning offer by brainstorming regarding the teacher's response to the MPSKT learned and then guiding them to the expected knowledge. Thus, the tasks are very important to include as learning resources that we set in the training model. We employed some types of tasks in this model, i.e. problem-solving tasks, problem-posing tasks, and context-based task, each taken either from math competition problems, tasks such as problem designed by the authors, and PISA mathematics problem, PISA-like problem, and journal article.

The following are the learning sequences we would describe in this paper in order.

Table 2. Learning sequences offered in the teacher training

Learning Sequence	Description	Time allocation (hours)
Teacher beliefs in building teachers' awareness toward how to teach mathematics learning and how students should learn mathematics		4
Problem solving	meaning of mathematical problem, types of mathematical problem: open-ended, context-based problem, problem-solving process and strategies, students as problem solvers	7
Problem posing	problem posing technique posing mathematical task in practice	3

Problem Reformulation (PS task with a given problem)	posing PS task related based on a given PS task	4
Problem Generation 1 (PS task with a given context)	posing PS task from a set of given information of context situation (group)	4
Problem Generation 2 (PS task employing self-chosen context)	posing PS task from context situation created/chosen by teachers (group)	4
Problem Generation 3 (task employing context from outdoor-field experience)	posing PS task from context situation created by teachers within outdoor-field activities	7
Instructional practices of PS	Peer teaching by using PS tasks posed by teachers	7
Reflection	discussing MPSKT based on problem solving-posing experiences as well as further recommendation related to teachers' responses	2

Table 2 shows the learning sequences offered in the training program differs in terms of the main content focused and the time allocation provided. The main focus in the training program at Mojokerto and Jember is PS content knowledge: types of mathematical problem, PS model and strategies, problem posing, while there was additional focus on the training program at the university, which is instructional practices for PS. Affective factors and beliefs were learned through the course of teacher beliefs in mathematics learning and teaching; PSCK was learned through the course of problem solving and problem posing; generation and reformulation, while PS pedagogical knowledge was learned through the course of instructional practice of PS.

Typically, each program took 7 hours per day. The program at Jember city and Mojokerto city each took 5 days, while the program at Surabaya city only took 2 days. Jember program did not carry out problem generation 3, while Mojokerto program did not carry out Instructional practices of PS due to limited time provided. On the other hand, Surabaya program only carried out 'teacher beliefs', 'problem-solving' and 'instructional practice for PS'. All of the programs were designed as workshops in which teachers were encouraged to work on the tasks provided within the programs. For example, when learning about problem-solving strategies, the facilitators asked teachers to work out particular problem-solving tasks with their own strategies, discuss their strategies regarding validating the strategies and if possible extending strategies. These all programs were guided by facilitators consisting of the authors and other invited persons who have been involved in some problem-solving-related training for teachers.

Results

Teachers' belief about mathematical learning and teaching

This course was carried out in all the three training programmes. The course started on the topic of teacher beliefs on structuring instructional sequence for helping students solve a mathematical problem. The facilitators guided teacher to discuss a task discussing the best steps in the instructional sequence. The teachers were asked to choose one of the best instructional sequences among three options, which respectively depict three views aligned with Ernest's (1989) view of the nature of mathematics: Instrumentalist, Platonist, and Problem-solving. In the training program at Surabaya, for example, teachers give various responds on this task. Some examples of responds are follows. The Instrumentalist view indicated from teachers reveal, "in this option, students first learn to construct early knowledge of something simple so easy to understand students, then gradually students learn more complex knowledge before solving the desired problem. By understanding the initial knowledge, students will be more easily to solve the problem." Meanwhile, the Platonist view argue, "I prefer to choose the third option, because the students were given clear steps on what they should do through the guidance provided in the worksheet. However, the teacher still keep students solve the problem in active discussion with their peers." Problem-solving view, on the other hand, is indicated as follows, "In option 2, learning has applied Polya step, because in solving the problem should students use Polya step from understanding the problem to looking back. In addition, it is clear there is a role of teachers in guiding students at every stage Polya done. As such, there were three types of learning sequences preferred by teachers. First, teachers start from demonstrating concepts/procedures, giving examples related to the concepts being learned, giving extra examples through real life task, asking students work on some exercises, guiding students work on such exercises, and finally giving extra task which are more complex. Second, teachers start from presenting some examples of solving problem related to particular concepts/procedures, asking students work out a problem-solving task in group through a worksheet in which the steps are provided to help students, guiding students solve the task, asking students present their strategies, and finally giving some extra exercises. Third, teachers start from introducing a problem-solving task, guiding students understand the problem of the task-devising strategies-carrying out the plan-and checking the results, encouraging students to find concepts related to the task being solved, and finally solve a more complex problem-solving task.

Problem solving content knowledge: nature of problem, model of PS, PS strategies, problem posing

In discussing the nature of problems, teachers were provided with 4 tasks and were asked to argue whether each of these tasks are problem or not for their students. The results of class discussions show that while the majority of teachers agreed that a mathematics problem should be interesting, challenging, and has no readily available procedure for finding the solution, the majority of teachers at all the training programmes assume that a math problem also a task that makes students difficult to solve since there is no any prerequisite knowledge held by students. This is evidenced by one of the student responses to the problem of straight line equations, which is actually suitable for secondary students, is a problem for elementary students. Next, the discussion is continued with the types of

mathematical problems. In this case the class discusses the characteristics of open problems and learned how to reformulate a closed-problem to a more open-ended problem. Thus, teachers were asked to work on the task below (Figure 2).

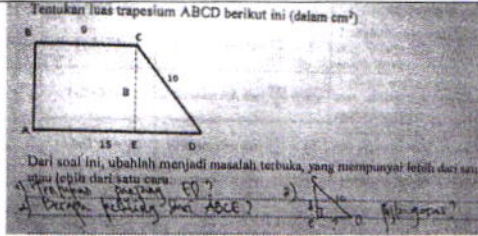
	Translation: Look at the task below Find the area of trapezoid ABCD below. (the figure) Make a new task by reformulating the task above so that it has more than one answer and/or one solution. Answer: 1) determine the length FD, 2) find the perimeter of ABCE
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Figure 2. Teachers' response of the task about open-ended problem

Examples of other responses found in the work of teachers are: "How many kinds of plane you see in the figure?", calculate the area of each of smaller planes in the figure, calculate the area of ABCD by applying Pythagoras formula." Although the majority of teachers have constraints to compose an open-ended problem, some other teachers have succeeded in composing open-ended issues that seem simple but open up opportunities for student creativity. This problem is like, "Suppose AB and BC are not known, specify the size AB and BC so that the area of trapezoid ABCD is 96 cm^2 ." The next discussion is about PS models. When discussing what steps a solver should carry out to solve a mathematics problem, the majority of teachers present the steps that a problem solver should take which is in line with Polya's (1973) steps, namely (1) understanding the problem, (2) preparing the plan / settlement strategy, (3) executing the settlement plan, and (4) check again. However, when discussing whether the sequence of mentioned steps of completion can always be used to solve other problems, the subject gives a uniform response, i.e. 'not always' for various reasons. The reasons, for examples, depend on the complexity of problem, which means the more complex (level of difficulty, amount of information) is a problem, a more unclear the steps of completion to solve such problem. Thus, they argued, it needs sometimes a number of repetition of certain stages of completion. This latter view is the closest to the concept of solving dynamic problems as proposed by Mason et al (1985), where it is quite possible that cyclical repetition occurs in problem-solving steps. The next discussion is related to problem solving strategies. The teachers were given by the task shown at figure 3, then were asked to identify what strategies the student is using to solve the problem and also to formulate other possible strategies.

An elementary student shows the result of the sum of $1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 + 10 + 11 +$

12 + 13 + 14 + 15 numbers by first making the table as follows.

Pattern	How is the pattern obtained?	Note
$1+2=3$	3 is obtained from $\frac{2 \times 3}{2} = 3$	2 is the last number and 3 is its subsequent number
$1+2+3=6$	6 is obtained from $\frac{3 \times 4}{2} = 6$	3 is the last number and 4 is its subsequent number
$1+2+3+4=10$	10 is obtained from $\frac{4 \times 5}{2} = 10$	4 is the last number and 5 is its subsequent number

From this table, she conclude that the sum of $1+2+3+\dots+15 = \frac{15 \times 16}{2} = 120$

Figure 3. Task for problem-solving strategies

In responding the task above, teachers identified some strategies, i.e. by summing all the number manually, by summing all the number in pairs, such as $1+15$, $2+14$, and so on, and then dividing the results by two as what a mathematician, Gauss, did, by applying formula of arithmetic series, and by counting strategies. However, when asked to determine which one is the most effective, the discussion leads the teachers argue that the strategy shown in the figure 3 is the most effective compared to the strategies they have set. This is because the strategy is considered simpler, just look at the pattern without counting or even calculating.

Problem posing: Problem generation and problem reformulation

In this session, teachers were encouraged to pose two types of problems collaboratively in group at problem generation 1-2 and problem reformulation 3, and individual at problem generation 3. In problem generation teachers were given a figure of ferris wheel which might be interesting as source of posing a fruitful mathematical problem.

The teachers worked in groups comprising 4-5 teachers. First, they solved the problem in problem reformulation and then continued with designing as many as problems that might be posed by reformulating the existing problem. We expected them to design mathematical problem for students which satisfy the characteristics problem based on their experiences when following problem-solving activities. Once they finished, we encouraged them to present their work in class discussion. The following is the mathematics problem designed and discussed during the class discussion.

The teachers work in groups of 4-5 teachers. They started from solving the problem of reformulation and then proceed to design as many problems as possible by reformulating the problem they have solved. We expected them to design mathematical problems based on their understanding of the nature of mathematics problem as well as problem-solving strategies in the previous learning. Once completed, we asked them to present their work in class discussions. Figure 4 exemplify one of problem designed and presented in the class discussion.

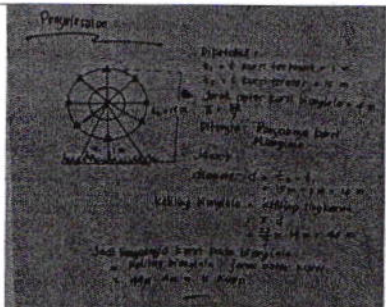
2. SOAL PROBLEM POSEING Di tengah kota baru, terdapat perantara "Ferris Wheel". Jika tinggi kabin pada posisi paling atas dari permukaan tanah 15 meter, dan tinggi kabin pada permukaan tanah 1 meter dari permukaan tanah. Tentukan: Berapa kabin yang terdapat pada Ferris Wheel apabila jarak antar kabin yang sama dengan yang lain adalah 4 meter. (diketahui $\pi = \frac{22}{7}$)	
In the center of city, there is a ferris wheel. The highest cabin has the height of 15 meter, and the lowest cabin is 1 meter above the ground. Determine the number of cabins if the distance between one cabin to another cabin is 4 meter. (given that $\pi = \frac{22}{7}$)	Answer: diameter: $d = t_2 - t_1 = 15 \text{ m} - 1 \text{ m} = 14 \text{ m}$ Circumference = $\pi \cdot d = \frac{22}{7} \cdot 14 \text{ m} = 44 \text{ m}$ So, the number of cabin of the ferris wheel is $44 \text{ m} : 4 \text{ m} = 11 \text{ m}$

Figure 4. Problem designed by teachers in problem reformulation

In this discussion session, we found that teachers at this stage had problem of posing problem-solving task regarding insufficient information. This is indicated from the teachers' discussion at the task posed by teacher (Figure 4) which does not clearly state which distance that is meant in the provided information. The phrase 'distance between one cabin to another does not make mathematical sense' which might cause misinterpretation among solvers. Finally, we found some points of reflection, i.e. insufficient information, context, authenticity, language structure, and limited strategies potentially emerged from the posed problem, and mathematically implausible problem. These points were then used as a reflection for the teachers in the subsequent problem posing activities.

Problem solving pedagogical knowledge: PS instructional sequence

In this session, teachers were encouraged to organize teaching practice in the classroom where each of group of teachers asked one of their members to act as a teacher and other teachers as students. The teaching practices utilized problems resulted by each group in the problem generation session. Besides acting as students, the other teachers also become observers which examine the quality of teaching carried out by the teacher. The results of observation were then presented and discussed after the teaching practice. The facilitators guide teachers to observed the presenting group of teachers by a examining how such teacher guide students (the other teachers) solve the designed problem from the stage of understanding problem to looking back the solutions of the problem.

Discussion and Conclusion

The learning sequences we designed in the training model were developed around the view of innovative approaches applied to a teacher professional development. Such approaches follow a view that an innovative professional development should focus on the construction of participants' knowledge through the circuitous routes taking place within the context of teachers' experience (Campbell, 2012). We argue there are at least four aspects which makes this model is innovative. First, the model supports teachers' understanding of MPSKT by employing initial teachers' experience and knowledge of the certain topic of MPSKT as sources of learning, aside from the lesson delivered by facilitators. For instance, when learning problem reformulation, the 'mathematics problem' designed by teachers were still far below from the standard of a good problem-solving task, in which plausibility, language structure, and authenticity of the problem context become main obstacles. This experience was then discussed in class by encouraging teachers to give some reviews on whatever they think about the problem posed by other teachers. As such, facilitators tried to summarize and clarify the findings with some theoretical perspectives of the standard of a good problem-solving task through interactive discussion as the reflection for subsequent learning sequences. Second, the professional learning course offered in this model was developed around views of problem solving and problem posing activities. This is respect to the findings that problem-solving activities could help teachers understand mathematics content knowledge, thereby, Singapore teacher education curriculum puts problem-solving as major approach for teachers to learn such knowledge (Wong, Boey, Lim-Teo & Dindyal, 2012). In addition, such knowledge can also be enhanced through problem-posing activities (Toluk-Uçar, 2009). Such view is supported by the study of Kwek (2015) reporting that thinking habits within problem posing not only enhanced problem-solving skills but also helped to reinforce and enrich basic mathematical concepts. Thus, both problem-solving and problem posing activities are expected to enhance teachers' understanding of MPSKT. Third, the model implements an outdoor field experience for problem generation 3. Within this activity, teachers were encouraged to create a problem-solving task in an outdoor field experience based on context situation they found. Such experience can help teachers to explore and understand the situation around them and help them feel more connected to their natural world (Moss, 2009), thus, teachers become more sensitive toward the possible context that can be generated into a problem-solving task. Therefore, we argue that teachers would be able to 'digest' a variety of context situation around them, choose the best context, and translate it into a problem-solving task with the authentic context they want to design. Siswono et al (2018) found that this activity could elicit teachers' idea of developing context variability and the level use of context authenticity. Fourth, teachers experienced from solving problem-solving tasks, posing problem-solving task, to using the posed problem-solving task as a resource of learning in the peer teaching. Through this experience, teachers could generate a deeper self-reflection from the process they underwent so that it will be valuable for problem-solving experience, particularly, in their actual classroom setting.

The design principles of this present training teacher model also try to answer the challenges of providing professional development which focus on helping teachers learn

essential needs for teaching mathematics. Prior to the implementation of this training model, we collected teachers' responses of their experience in joining professional development program through a short interview of some participants. We found that the form of training program they have ever been involved most frequently is around discussing about the surface feature aspects of teaching mathematics, such as developing lesson plan whose components meet the standard mandated by ministry of education, the change of curriculum, evaluating students' mathematics achievement based on the current curriculum, and recognizing particular learning media. The essential aspects such as understanding the content of mathematics and how such contents are delivered to students, however, are scant to be discussed in their professional development program. This type of TPD, as some Indonesian studies reported (e.g. see Widodo & Riandi, 2013; Ekawati & Kohar, 2016), is usually top down with predetermined strategies and expected outcomes from the current curriculum regarding the topic of particular issues being learned. This condition is in line with the findings that many professional development are found irrelevant, ineffective, and for not giving teachers what they actually need to teach students (Marrero et al., 2010).

In the end of the training, the teachers were encouraged to reflect on the activities of training they joined. We asked them wrote their difficulties and impression of learning MPSKT in an open questionnaire. In relation to problem posing, we found that they had difficulties in finding authentic contexts and composing an open-ended problem. This finding become reflections for the training program as well. Also, since this is an initial design of professional development which support teachers' MPSKT, we are aware of limited courses which facilitate as many as aspects of MPSKT componenets as proposed by Chapman (2015). Our design primarily focus on problem-solving content knowledge, thus, the aspects of pedagogical problem-solving knowledge such as students as problem solver and instructional strategies for problem-solving are not yet main focus of our design. The course of 'instructional practices' presented in the results section only discuss the extend to which the teaching of the presenting teacher meet the characteristics of consultative teaching, in which student-centered teaching become main feature. However, other aspects mentioned by Chapman (2015) such as how to use of technology in problem-solving instruction, assessment of students' problem-solving progress, as well as when and how to intervene during students' problem-solving are not yet explicitly designed in the model. Therefore, a more comprehensive design of professional learning for teachers which considers all the components of MPSKT offered by Chapman (2015) need to be further developed.

Acknowledgement

This is the initial work of research in the research scheme of Strategi Nasional year 2018. We would like to thank Ministry of Higher Education and Research, Indonesia, for the grant, the rector of Universitas Negeri Surabaya, and the dean of Faculty of Mathematics and Natural Science.

References

- Campbell, T. (2012). Building community in triads involved in science teacher education: An innovative professional development model. *Brock Education Journal*, 21(2), 53-69.
- Chapman, O. (2015). Mathematics teachers' knowledge for teaching problem solving. *LUMAT (2013-2015 Issues)*, 3(1), 19-36.
- Ekawati, R., & Kohar, A. W. (2017). Innovative Teacher Professional Development within PMRI in Indonesia. *International Journal of Innovation in Science and Mathematics Education*, 24(5), 1-13.
- Ernest, P. (1989). The impact of beliefs on the teaching of mathematics. In P. Ernest (Ed.), *Mathematics teaching: The state of the art* (pp. 249-253). New York: Falmer.
- Kwek, M.L. (2015). Using Problem Posing as a Formative Assessment Tool. In Singer, F.M., Ellerton, N.F. & Cai, J. (Eds), *Mathematical Problem Posing: From Research to Effective Practice* (pp. 273-279). New York: Springer
- Marrero, M. E., Riccio, J. F., Woodruff, K. A., & Schuster, G. S. (2010). Live, online short-courses: A case study of innovative teacher professional development. *The International Review of Research in Open and Distributed Learning*, 11(1), 81-95.
- Mason J., Burton L., & Stacey K. (1985). *Thinking Mathematically*. Boston, MA: Addison-Wesley.
- Moss, M. (2009). Outdoor Mathematical Experiences: Constructivism, Connections, and Health. In Clarke, B., Grevholm, B., Millman, R. (Eds). *Tasks in Primary Mathematics Teacher Education* (pp. 263-273). Springer US.
- Mullis, I. V., Martin, M. O., Foy, P., & Arora, A. (2012). *TIMSS 2011 international results in mathematics*. International Association for the Evaluation of Educational Achievement. Herengracht 487, Amsterdam, 1017 BT, The Netherlands.
- OECD. (2015). *PISA 2015: Results in focus*. Retrieved from <https://www.oecd.org/pisa/pisa-2015-results-in-focus.pdf>
- Polya, G. (1973). *How to solve it: A new aspects of mathematical methods*. US: Prentice University Press.
- Schoenfeld, A. H. (1985). *Mathematical problem solving*. London: Academic Press.
- Silver, E. A. (1994). On mathematical problem posing. *For the learning of mathematics*, 14(1), 19-28.
- Siswono, T. Y. E., Kohar, A. W., Kurniasari, I., & Astuti, Y. P. (2016, February). An Investigation of Secondary Teachers' Understanding and Belief on Mathematical Problem Solving. In *Journal of Physics: Conference Series*, 693(1), 012015. IOP Publishing.
- Siswono, T. Y., Kohar, A. W., Rosyidi, A. H., & Hartono, S. (2017). Primary school teachers' beliefs and knowledge about mathematical problem-solving and their performance on problem-solving task. *World Trans. on Engng. and Technol. Educ*, 15(2), 126-131.
- Siswono, T. Y. E., Kohar, A. W., Rosyidi, A. H., & Hartono, S. (2018, January). Searching for Authentic Context in Designing PISA-like Mathematics Problem: From Indoor to

- Outdoor Field Experience. In *Journal of Physics: Conference Series* (Vol. 953, No. 1, p. 012197). IOP Publishing.
- Steinbring, H. (1998). Elements of epistemological knowledge for mathematics teachers. *Journal of Mathematics Teacher Education*, 1(2), 157-189.
- Tolak-Uçar, Z. (2009). Developing pre-service teachers understanding of fractions through problem posing. *Teaching and Teacher Education*, 25(1), 166-175.
- Widodo, A. & Riandi (2013) Dual-mode teacher professional development: challenges and re-visioning future TPD in Indonesia, *Teacher Development: An international journal of teachers' professional development*, 17(3), 380-392
- Wong, K. Y., Boey, K. L., Lim-Teo, S. K., & Dindyal, J. (2012). The preparation of primary mathematics teachers in Singapore: programs and outcomes from the TEDS-M study. *ZDM*, 44(3), 293-306.
- Zaslavsky, O (2009). Mathematics educators' knowledge and development In R Even & DL Ball (eds), *The Professional Education and Development of Teachers of Mathematics* (pp 105-111), US: Springer.

[Tatag Yuli Eko Siswono]

[Universitas Negeri Surabaya, Gedung D1 FMIPA Kampus Ketintang Unesa, Surabaya, 60231]

[tatagsiswono@unesa.ac.id]

[Ahmad Wachidul Kohar]

[Universitas Negeri Surabaya, Gedung D1 FMIPA Kampus Ketintang Unesa, Surabaya, 60231]

[ahmadw.kohar@gmail.com]

[Sugi Hartono]

[Universitas Negeri Surabaya, Gedung D1 FMIPA Kampus Ketintang Unesa, Surabaya, 60231]

[sugihartonounesa@gmail.com]

[Abdul Haris Rosyidi]

[Universitas Negeri Surabaya, Gedung C8 FMIPA Kampus Ketintang Unesa, Surabaya, 60231]

[abdulharis@unesa.ac.id]